

Derivatives of Logarithmic and Exponential Functions of Any Base

Formulas: These formulas should be memorized.

Derivative of the exponential function with a base e : $\frac{d}{dx} e^x = e^x$

Derivative of an exponential function with any base a : $\frac{d}{dx} a^x = a^x \ln a$

Derivative of the natural logarithm (base e): $\frac{d}{dx} \ln x = \frac{1}{x}$

Derivative of a logarithm of any base a : $\frac{d}{dx} \log_a x = \frac{1}{x \ln a}$

Notice that the derivative of a logarithm base a can also be found using the change of base formula, $\log_a x = \frac{\ln x}{\ln a}$, then applying the formula for the derivative of the natural logarithm.

Exercises: Find the indicated first or second derivative.

1. $g(x) = e^x$. Find $g'(x)$.
2. $y = 2^t$. Find $\frac{dy}{dt}$.
3. $f(x) = 3^x + \frac{1}{11}$. Find $f''(x)$.
4. Find $\frac{d}{du} (5e^u - 0.5^u)$
5. $h(x) = \ln \pi + \ln x$. Find $h'(x)$.
6. $y = \log t$. Find $\frac{dy}{dt}$.
7. $f(x) = \log_3 x$. Find $f''(x)$.
8. Find $\frac{d}{du} (5 \ln u - \log_{0.5} u)$
9. $D_x(7e^x - \log_7 x)$
10. $y = 4^x + 4x^4$. Find $\frac{d^2y}{dx^2}$.
11. Find $\frac{d}{du} \left(7 \cdot 3^u - \frac{u}{7} + 5 \log_3 u \right)$
12. $f(x) = x \log_3 x$. Find $f'(x)$.
13. Find $\frac{d^2}{du^2} (5ue^u)$
14. $g(x) = \frac{x^2}{\log_2 x}$. Find $g'(x)$.
15. $y = x^2 4^x + \frac{1}{\ln x}$. Find $\frac{dy}{dx}$.

Answers

1. $g'(x) = e^x$
2. $\frac{dy}{dt} = 2^t \ln 2$
3. $f''(x) = 3^x (\ln 3)^2$
4. $5e^u - 0.5^u \ln 0.5$ or $5e^u + 0.5^u \ln 2$
5. $h'(x) = \frac{1}{x}$
6. $\frac{dy}{dt} = \frac{1}{t \ln 10}$
7. $f''(x) = -\frac{1}{x^2 \ln 3}$
8. $\frac{5}{u} - \frac{1}{u \ln \frac{1}{2}}$ or $\frac{5}{u} + \frac{1}{u \ln 2}$
9. $7e^x - \frac{1}{x \ln 7}$
10. $\frac{d^2y}{dx^2} = 4^x (\ln 4)^2 + 48x^2$
11. $7 \cdot 3^u \ln 3 - \frac{1}{7} + \frac{5}{u \ln 3}$
12. $f'(x) = \frac{1}{\ln 3} + \log_3 x$ or $f'(x) = \frac{1 + \ln x}{\ln 3}$
13. $5ue^u + 10e^u$
14. $g'(x) = \frac{2x(\ln 2)(\log_2 x) - x}{\ln 2(\log_2 x)^2}$ or $g'(x) = \frac{x \ln 2(2 \ln x - 1)}{(\ln x)^2}$
15. $\frac{dy}{dx} = x^2 4^x \ln 4 + 2x 4^x - \frac{1}{x (\ln x)^2}$