Modules D and E Sample Test Items
MGF1106 Foundations of Mathematical Reasoning

VOCABULARY. Fill in the blank with the correct term or phrase. Select from this list of terms.

| permutation | combination | factorial | uniform |
| :--- | :--- | :--- | :--- |
| product table | tree diagram | sample sapce | empirical |
| complements | theoretical | additive | conditional |
| impossible | guaranteed | likely | Fundamental Counting Principle |
| independent | unlikely | dependent | mutually exclusive |

1) In counting, a(n) $\qquad$ can be used to analyze a two-part task.
2) The $\qquad$ of an experiment is the set of all possible outcomes of the experiment.
3) A multiple-part task is said to be $\qquad$ if the number of choices for any particular part of the task is the same no matter which choices were selected for previous parts.
4) The $\qquad$ probability of an event $E$ is the number of favorable outcomes divided by the total number of possible outcomes.
5) When a task consists of multple parts and satisfies the uniformity criterion, the tells us that we can count the number of ways to complete the task by multiplying the number of ways to complete each part of the task.
6) The $\qquad$ principle of counting is that the number of ways a certain condition can be satisfied is the total number of possible results minus the number of ways the condition would NOT be satisfied.
7) The $\qquad$ probability of an event $E$ is the number of times the event $E$ occured divided by the number of times the experiment was performed.
8) The $\qquad$ priciple of counting is that the number of ways that one OR the other of two conditions could be satisfied is the number of ways one could be satisfied plus the number of ways the other could be satisfied minus the number of ways they could both be satisfied together.
9) $A(n)$ $\qquad$ of a group of objects is an arrangement of those objects where order is important.
10) When two events, $A$ and $B$, are $\qquad$ , the probability of $A$ and $B$ occuring is the probability of $A$ multiplied by the probability of $B$.
11) When two events, $A$ and $B$, are $\qquad$ , the probability of $A$ and $B$ occuring is 0 .
12) $A(n)$ $\qquad$ event has a probability of 0 .
13) The $\qquad$ of a natural number n is found by multiplying all the natural numbers less than or equal to $n$.
14) $A(n)$ $\qquad$ of a group of objects is a subset of those objects, where order is not important.

## MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

15) How many committees of four people can be formed from a group of ten people?
A) 3
B) 210
C) 2
D) $3,628,800$
16) A security team visites 8 offices each night. How many different ways can the team order its visits?
A) 64
B) 8
C) 40,320
D) $16,777,216$

Determine the number of figures (of any size) in the design.

A) 13
B) 12
C) 17
D) 18

## Determine whether the events are independent.

18) A bag contains 9 red and 8 green marbles. A marble is drawn, replaced in the bag, then a second marble is drawn. Are the events "first marble is red" and "second marble is green" independent events?
A) No
B) Yes

Evaluate the expression.
19) $14 \mathrm{C}_{8}$
A) $60,540,480$
B) 1440
C) $2,162,160$
D) 3003

Evaluate the factorial expression. 20) 4 !
A) 12
B) 48
C) 6
D) 24

Evaluate the permutation.
21) ${ }_{8} \mathrm{P}_{2}$
A) 1
B) 336
C) 56
D) 8

Find the conditional probability.
22) If a single fair die is rolled, find the probability that the number rolled is 5 given that it is odd.
A) $\frac{1}{3}$
B) $\frac{1}{2}$
C) $\frac{1}{6}$
D) $\frac{2}{3}$

## Find the indicated probability.

23) An unprepared student makes random guesses for the ten true-false questions on a quiz. Find the probability that there is at least one correct answer.
A) $\frac{1}{1024}$
B) $\frac{1023}{1024}$
C) $\frac{9}{10}$
D) $\frac{1}{10}$
24) The table below shows the soft drinks preferences of people in three age groups.

|  | cola | root beer | lemon-lime |
| ---: | :---: | :---: | :---: |
| under 21 years of age | 40 | 25 | 20 |
| between 21 and 40 | 35 | 20 | 30 |
| over 40 years of age | 20 | 30 | 35 |

If one of the 255 subjects is randomly selected, find the probability that the person is over 40 years of age given that they drink root beer.
A) $\frac{6}{17}$
B) $\frac{5}{17}$
C) $\frac{2}{5}$
D) None of the above is correct.
25) The table shows the distribution of family size in a certain U.S. city

| Family Size | Probability |
| :---: | :--- |
| 2 | 0.450 |
| 3 | 0.209 |
| 4 | 0.201 |
| 5 | 0.095 |
| 6 | 0.029 |
| $7+$ | 0.016 |

A family is selected at random from the city. Find the probability that the size of the family is between 2 and 5 inclusive. Round approximations to three decimal places.
Hint: This means that the family is of size 2 or 3 or 4 or 5 .
A) 0.41
B) 0.545
C) 0.955
D) 0.86

## Find the number of ways to get the following card combinations from a 52-card deck.

26) Two black cards and three red cards
A) $\left(26 \mathrm{C}_{2}\right)\left(26 \mathrm{C}_{3}\right)=845,000$ ways
B) $26 \cdot 25 \cdot 26 \cdot 25=422,500$ ways
C) $26 \cdot 26 \cdot 26 \cdot 26 \cdot 26=11,881,376$ ways
D) $\left(26 \mathrm{P}_{2}\right)\left(26 \mathrm{P}_{3}\right)=10,140,000$ ways

## Find the probability.

27) If you are dealt two cards successively (with replacement of the first) from a standard 52-card deck, find the probability of getting a face card on the first card and an ace on the second.
A) $\frac{12}{52} \times \frac{12}{52}=\frac{9}{169}$
B) $\frac{12}{52} \times \frac{4}{52}=\frac{3}{169}$
C) $\frac{13}{52} \times \frac{12}{51}=\frac{1}{17}$
D) $\frac{12}{52} \times \frac{4}{51}=\frac{4}{221}$
28) A spinner has regions numbered 1 through 21 . What is the probability that the spinner will stop on an even number or a multiple of 3 ?
A) $\frac{10}{9}$
B) 17
C) $\frac{2}{3}$
D) $\frac{1}{3}$

Give the probability that the spinner shown would land on the indicated color.
29) black

A) $\frac{1}{6}$
B) $\frac{1}{3}$
C) $\frac{1}{2}$
D) $\frac{2}{3}$

Given a group of students: G = \{Allen, Brenda, Chad, Dorothy, Eric\} or $\mathbf{G}=\{\mathbf{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}, \mathrm{E}\}$, list and count the different ways of choosing the following officers or representatives for student congress. Assume that no one can hold more than one office.
30) A president, a secretary, and a treasurer, if the president must be a woman and the other two must be men.
A) BAC, BAE, BCE, DAC, DAE, DCE, BCA, BEA, BEC, DCA, DEA, DEC; 12
B) $\mathrm{ABD}, \mathrm{CBD}, \mathrm{EBD} ; 3$
C) BAC, BAE, DAC, DAE; 4
D) BAC, BAE, BCE, DAC, DAE, DCE; 6

If two fair dice, one red and one white, are rolled, in how many ways can the result be obtained?
31) The sum of the two dice is less than 5 .
A) 5 ways
B) 4 ways
C) 6 ways
D) 10 ways

## Provide an appropriate response.

32) Consider determining how many possible phone numbers are within a particular area code (repeated numbers allowed). Is this a combination, a permutation, or neither?
A) Neither
B) Combination
C) Permutation
33) Consider the following counting problem. A pool of possible jurors consists of 11 men and 13 women. How many different juries consisting of 5 women and 7 men are possible?

To solve this problem, which of the following rules would you use?
A) Both the permutations rule and the fundamental counting principle
B) Both the combinations rule and the fundamental counting principle
C) The combinations rule only
D) The fundamental counting principle only

Solve the problem involving probabilities with independent events.
34) A single die is rolled twice. Find the probability of getting a 1 the first time and a 4 the second time.
A) $\frac{1}{12}$
B) $\frac{1}{3}$
C) $\frac{1}{36}$
D) $\frac{1}{6}$

## Solve the problem.

35) How many ways can a president, vice-president, secretary, and treasurer be chosen from a club with 10 members? Assume that no member can hold more than one office.
A) 40
B) 24
C) 210
D) 5040
36) License plates are made using 3 letters followed by 2 digits. How many plates can be made if repetition of letters and digits is allowed?
A) 100,000
B) 175,760
C) $11,881,376$
D) $1,757,600$
37) How many 3-digit numbers are there in our system of counting numbers?
A) 900
B) 27
C) 899
D) 6
38) A shoe store carries one brand of shoe in 3 different styles, 4 sizes, and 5 colors. How many different shoes are available of this one brand?
A) 12
B) 35
C) 60
D) 36
39) At a lumber company, shelves are sold in 4 types of wood, 3 different widths and 3 different lengths. How many different types of shelves could be ordered?
A) 10
B) 21
C) 36
D) 48
40) How many three-digit counting numbers are even?
A) $10 \cdot 10 \cdot 4=400$ numbers
B) $10 \cdot 10 \cdot 5=500$ numbers
C) $9 \cdot 10 \cdot 4=360$ numbers
D) $9 \cdot 10 \cdot 5=450$ numbers
41) A basket contains 6 oranges and 4 tangerines. A sample of 3 is drawn. Find the probability that they are all oranges.
A) $\frac{1}{5}$
B) $\frac{1}{3}$
C) $\frac{1}{6}$
D) $\frac{4}{9}$
42) Mr. Larsen's third grade class has 22 students, 12 girls and 10 boys. Two students must be selected at random to be in the fall play. What is the probability that no boys will be chosen? Order is not important.
A) $\frac{6}{11}$
B) $\frac{5}{6}$
C) $\frac{2}{7}$
D) $\frac{1}{6}$
43) The table shows the number of college students who prefer a given pizza topping.

| toppings | freshman sophomore | junior | senior |  |
| ---: | ---: | ---: | ---: | ---: |
| cheese | 16 | 16 | 21 | 28 |
| meat | 24 | 28 | 16 | 16 |
| veggie | 16 | 16 | 24 | 28 |

Find the empirical probability that a randomly selected student prefers cheese toppings.
A) 0.325
B) 0.346
C) 0.112
D) 0.337
44) Construct a product table showing all possible two-digit numbers using digits from the set $\{1,2,6,7\}$.
A)

| 1 | 2 | 6 | 7 |  |
| :--- | :--- | :--- | :--- | :--- |
| 1 | 111121617 |  |  |  |
| 2 | 21 | 22 | 26 | 27 |
| 6 | 61 | 6266 | 67 |  |
| 7 | 71 | 727677 |  |  |

B)

| $\mid 1 \quad 2$ |
| :--- |
| $6 \mid 6162$ | 77172

C)

| 1 | $2 \quad 6 \quad 7$ |
| :--- | :--- | :--- | :--- |
| 1 | 11216171 |
| 2 | 12226272 |
| 6 | 16266671 |
| 7 | 17276777 |

D)
$\begin{array}{r}1267 \\ \hline 12378\end{array}$
23489
6781213
7891314

## Use counting rules to determine the probability.

45) A committee of 9 members is voting on a proposal. Each member casts a yea or nay vote. On a random voting basis, what is the probability that the final vote count is unanimous (either all yea, or all nay)?
A) $\frac{1}{72}$
B) $\frac{1}{256}$
C) $\frac{1}{512}$
D) $\frac{1}{492}$

Use the general multiplication rule to find the indicated probability.
46) You are dealt two cards successively (without replacement) from a shuffled deck of 52 playing cards. Find the probability that both cards are black.
A) $\frac{13}{51}$
B) $\frac{25}{51}$
C) $\frac{1}{2652}$
D) $\frac{25}{102}$

Using the product table for rolling two dice, one red and one green, list and count the outcomes for which the sum (for both dice) is the following.
47) Greater than 10

|  |  | Green Die |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 1 | 2 | 3 | 4 | 5 | 6 |  |
| Red <br> Die | 1 | $(1,1)$ | $(1,2)$ | $(1,3)$ | $(1,4)$ | $(1,5)$ | $(1,6)$ |  |
|  | 2 | $(2,1)$ | $(2,2)$ | $(2,3)$ | $(2,4)$ | $(2,5)$ | $(2,6)$ |  |
|  | 3 | $(3,1)$ | $(3,2)$ | $(3,3)$ | $(3,4)$ | $(3,5)$ | $(3,6)$ |  |
|  | 4 | $(4,1)$ | $(4,2)$ | $(4,3)$ | $(4,4)$ | $(4,5)$ | $(4,6)$ |  |
|  | 5 | $(5,1)$ | $(5,2)$ | $(5,3)$ | $(5,4)$ | $(5,5)$ | $(5,6)$ |  |
|  | 6 | $(6,1)$ | $(6,2)$ | $(6,3)$ | $(6,4)$ | $(6,5)$ | $(6,6)$ |  |

А) $(4,6),(6,4),(5,6),(6,5),(5,5) ; 5$
B) $(6,5),(5,6),(6,6) ; 3$
C) $(6,6) ; 1$
D) $(6,5),(6,6) ; 2$

FREE RESPONSE. Show your work for full credit. Partial credit will be given for statement of related formulas, or for organizing work using techniques presented in class. Solution to application exercises must be detailed to receive full credit, including the steps for any approach mentioned in the instructions or covered in class.

Use a tree diagram showing all possible results when four fair coins are tossed. Then list the ways of getting the indicated result.
48) exactly two tails

1) product table
2) sample space
3) uniform
4) theoretical
5) Fundamental Counting Principle
6) complements
7) empirical
8) additive
9) permutation
10) independent
11) mutually excusive
12) permutation
13) factorial
14) combination
15) B
16) C
17) C
18) $B$
19) $D$
20) D
21) C
22) $A$
23) $B$
24) C
25) C
26) $A$
27) B
28) C
29) $B$
30) A
31) C
32) $A$
33) $B$
34) C
35) D
36) D
37) A
38) C
39) C
40) D
41) C
42) C
43) $A$
44) A

Testname: COUNTING AND PROBABILITY SAMPLE TEST QUESTIONS
45) B
46) D
47) B
48) hhtt, htht, htth, thht, thth, tthh


